Loss Calculation of Single and Coupled Strip Lines by Extended Spectral Domain Approach

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Abstract—**The extended spectral domain approach is used to calculate the losses of the strip lines of arbitrary thickness. This versatile method is applicable to single and coupled strip lines with isotropic and/or anisotropic substrates. Both the simpler quasistatic and the rigorous hybrid-mode formulation can be developed on the same basis. Numerical examples presented include loss calculations of the strip lines with thick, as well as thin conductors, where thickness is comparable to or less than the skin depth. Numerical results also reveal the usefulness and the limitation of the quasistatic analyses.**

I. Introduction

The EXTENDED spectral domain approach (ESDA) is quite versatile and can be used to analyze the propagation characteristics of various types of planar transmission lines [1]–[4]. The approach has been applied to open as well as to shielded structures with multiconductors and stratified isotropic and/or anisotropic media [2], [4]. Both the quasistatic and hybrid-mode formulation can be developed on the same basis [1]–[4]. Also the procedure can take the conductor thickness into consideration, and therefore it can be extended easily to evaluate the conductor loss. Loss calculations based on techniques assuming zero metallization thickness, have caused some computational difficulty in the past [5], [6]. The present procedure can calculate the losses of single and coupled strip lines without any assumption of the conductor thickness.

II. Formulation by Extended Spectral Domain Approach

In ESDA, the electromagnetic fields can be expressed in terms of aperture fields. That is, the fields in the hybrid-mode formulation are related to the electric field vectors at the aperture surfaces $e^U(x), e^L(x)$, (Fig. 1(a)) [1], [2]. In the quasistatic approximation, the electric fields are expressed in terms of the x-components of electric field $e_x^U(x), e_x^L(x)$ [3] and the magnetic fields are related to the y-components of the magnetic flux densities at the aperture surfaces $b_y^U(x), b_y^L(x)$ (Fig. 1(b)) [7].

Quasistatic values of phase constant B and characteristic impedance Z_0 can be expressed in terms of the line inductance and capacitance. The stationary expression of the capacitance can be derived from the electric field expression [4], while that of the inductance can be derived from the magnetic field expression [7]. The unknown aperture fields, $e_x^U(x), e_x^L(x), b_y^U(x)$, and $b_y^L(x)$ in the quasistatic analysis can be determined by applying the Ritz procedure or the Galerkin procedure [4], [7] to the stationary expressions.

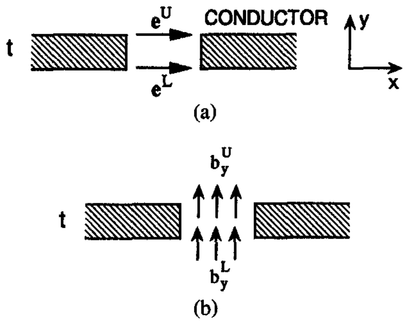

Fig. 1. Aperture fields. (a) Electric fields. (b) Magnetic flux densities.

Hybrid-mode analysis is developed by using the magnetic field representations expressed in terms of $\boldsymbol{e}^U(x)$, $\boldsymbol{e}^L(x)$. Applying the continuities of the magnetic fields at the aperture surfaces, we obtain the integral equations for the aperture fields $\boldsymbol{e}^U(x)$, $\boldsymbol{e}^L(x)$ and implicitly the phase constant β. Then, applying Galerkin's procedure [1]–[4] to the integral equations, we obtain the determinantal equation for β.

In the quasistatic approximation, the incremental inductance formula [8] has been utilized extensively for evaluating the loss due to the imperfect conductor. On the other hand, the perturbational procedure has been used widely for conductor loss calculations in the hybrid-mode analysis.

The incremental inductance formula requires the derivative of the inductance with respect to the normal to conductor surfaces [8],

$$\alpha_c = \frac{1}{2\mu_0 Z_0} \sum_i R_{si} \frac{\partial L}{\partial n_i}, \tag{1}$$

where R_s is the surface resistance of an infinitely thick conductor. The method is based on the assumption that the conductor thickness t is sufficiently greater than the skin depth $\delta(t > 3\delta)$, i.e., the losses on both surfaces of the conductor can be evaluated independently.

In this work, the perturbational procedure has been used to compute the conductor losses of strip lines both for the quasistatic and the hybrid-mode analyses, i.e., the attenuation due to the imperfect conductor is evaluated by

$$\alpha_c = \frac{P_C}{2P_0}, \tag{2}$$

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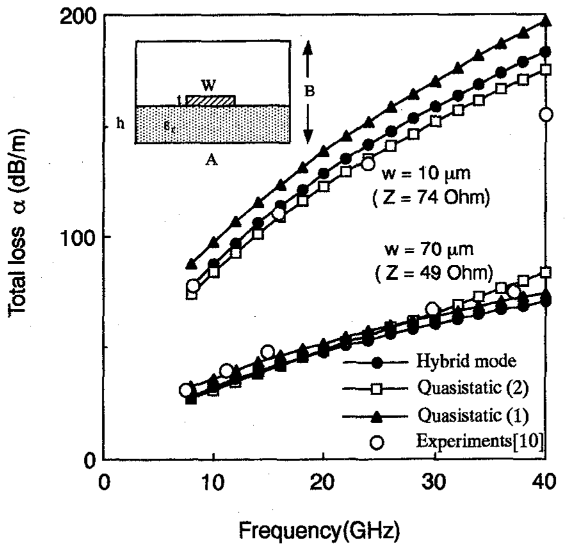

Fig. 2. Total losses of shielded strip line. Dielectrics: $\varepsilon_r = 12.9$, $\tan\delta = 0.0003$, $h = 0.1$ mm. Conductor: $\sigma = 4.1 \times 10^7$ S/m, $t = 3$ μm.

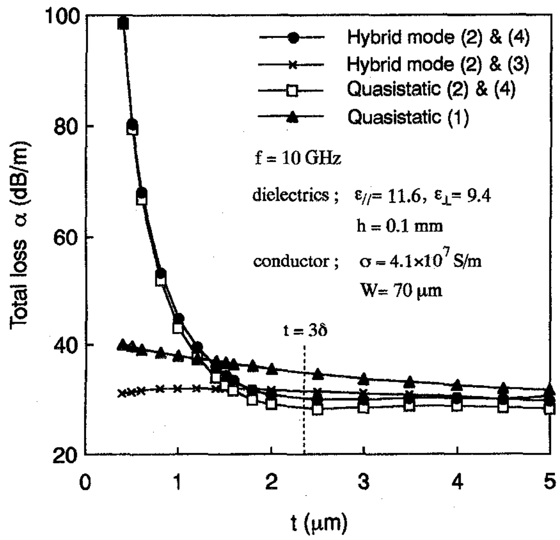

Fig. 3. Metallization thickness effect on attenuation.

where P_C is the power lost in the conductors. In the conventional perturbational methods, P_C has been calculated by the surface integral of the tangential component of the magnetic field $\boldsymbol{H}_t$ over the conductor surface C [6]

$$P_C = \frac{1}{2} R_s \int_C |\boldsymbol{H}_t|^2 dl. \tag{3}$$

For transmission lines with thin conductors ($t < 3\delta$), the fields penetrating from both surfaces of the conductor overlap each other, and the power loss P_C cannot be evaluated by the surface integral over C. Instead, P_C should be calculated by [9]

$$P_C = \frac{1}{2} \int_{S_c} \sigma |\boldsymbol{E}|^2 dS, \tag{4}$$

where S_C stands for the region occupied by the conductor. The electric field E inside conductors can be related to the tangential component of the magnetic field on the conductor surface $\boldsymbol{H}_t$ easily, and the integral over the conductor S_C can be reduced to an integral over the conductor surface C. The numerical computations involved require the integrals of the infinite Fourier integral or summation. Such integrations require enormous computation time. In ESDA, the orthogonality relation can be utilized advantageously to reduce double integrals to single integrals [6].

III. Numerical Examples

Fig. 2 shows the total (conductor and dielectric) losses for a shielded strip line. Loss due to imperfect dielectric is evaluated by the perturbational procedure [6]. The quasistatic values and the hybrid-mode values are presented in the figure. Also the measured values for the open strip line [10] are presented for comparison. The conductor thickness t of the numerical examples considered here is sufficiently greater than the skin depth $\delta (t > 3\delta)$. As seen in Fig. 2, the quasistatic values obtained by (1) and (2) are in good agreement with the hybrid-mode values for this case. Fig. 3 shows the metallization thickness effect on the attenuation constant of the strip line with the anisotropic sapphire substrate and realistic values of t. The quasistatic values based on the perturbational procedure (2) with (4) are in good agreement with the hybrid-mode values from (2) with (4) for both thin and thick metallizations, while the values determined by the incremental inductance formula (1), which assumes that conductor thickness t is sufficiently greater than the skin depth δ, are too low for the thin metallization. The minimum in α does not appear so clearly as that in [11] because of the smaller shape ratio W/h [12]. The figure includes the hybrid-mode values from (2) with (3) [6], which assume that $t >> d$ and become inaccurate for the thin metallization. Fig. 4 shows the total losses for coupled strip lines. It should be noted that the loss value of the odd-mode is larger than that of the even-mode. In the odd mode, the electromagnetic fields are concentrated between the strip conductors, which increases the current density near the edge and thus increasing the conductor loss. The quasistatic value by (2) with (4) give reasonable results for the odd-mode case over the whole frequency range, while in the even mode case the discrepancy seen between the quasistatic and the hybrid-mode becomes larger at the higher frequency range. As seen in Fig. 4, the frequency dependence of the effective dielectric constant of the even mode is larger than that of the odd mode. Furthermore, the field distribution of the even mode at the higher frequencies become so different from that of the quasistatic mode that the quasistatic loss calculations cannot be applied. A typical computation for the effective dielectric constant, the characteristic impedance, the conductor and dielectric loss with 10 basis functions takes approximately 3 minutes for the quasistatic analysis and 5 minutes for the hybrid-mode analysis on a 386 based personal computer.

IV. Conclusion

The extended spectral domain approach (ESDA) is used to calculate the losses of the strip lines. The procedure is

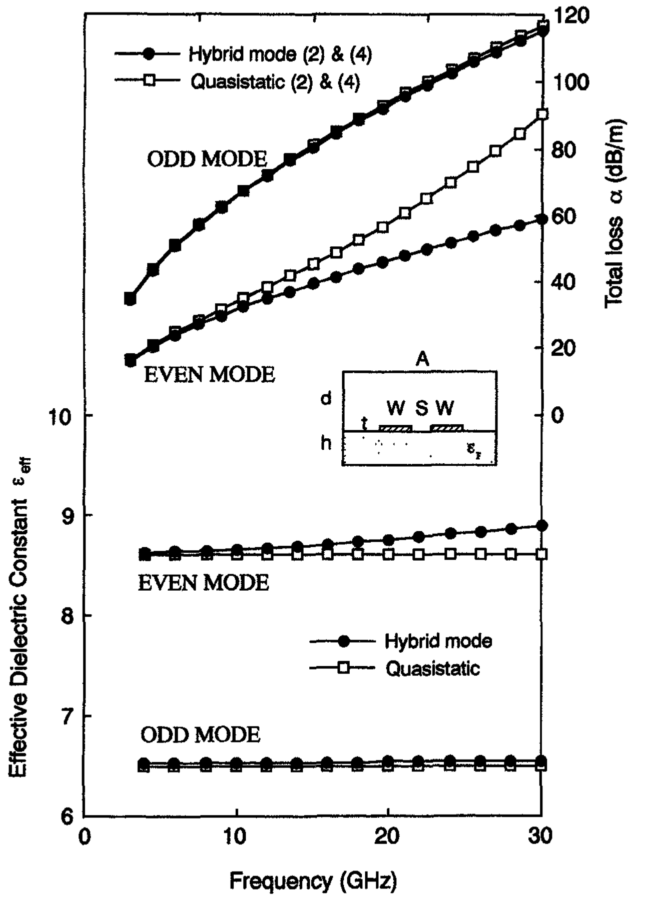

Fig. 4. Characteristics of coupled strip lines. Dielectrics: $\varepsilon_r = 12.9$, $\tan\delta = 0.0003$, $h = 0.1$ mm, $\sigma = 4.1 \times 10^7$S/m, $S = 30$ μm, $W = 70$ μm, $d = 2$ mm, $A = 1.2$ mm.

applicable to a wide range of the conductor thicknesses. Based on ESDA, both the simpler quasistatic and the rigorous hybrid-mode formulation can be developed for calculating losses. The substrate may be isotropic or anisotropic.

Numerical examples are compared with available measured data to show the validity of the method. Numerical examples include the losses of single strip lines and those of the even and odd modes of the coupled strip lines. The loss figures for lines with thin conductors, whose thickness is comparable to or less than the skin depth, are included. Numerical results also reveal the usefulness and the limitation of the quasistatic values.

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